

The Image Volume

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Abstract

The volume of an image can be defined by using the concept of vectors. Using this volume of an image a novel mathematical model has been developed for the first time. And this research the mathematical principles are used in very useful way. Research propose a new concept to find a volume of an image by using vector algebra. Volume of a parallelepiped is playing major role for the conceptual design part of this project. This concept can be used for practical applications such as in medical problems.

Key words: The volume of a parallelepiped

Introduction

An image gives much data some of which are hidden and others are visible. The focus in this paper is dealing with the hidden data of an image. As an example if we take a small object and we can determine the weight of the object by using the standard unit of measurement as shown figure 1. But if we take an image of that object we cannot quantify data of that image as shown in figure 2.



Figure-1

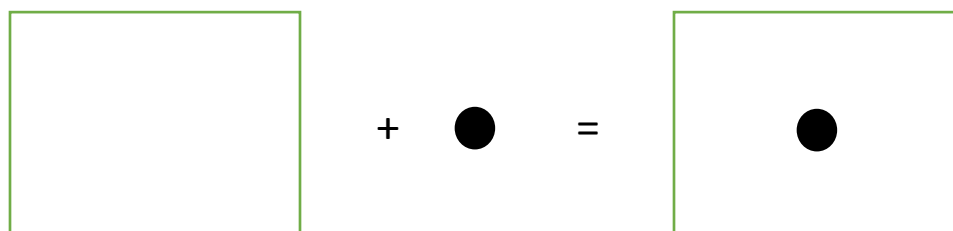
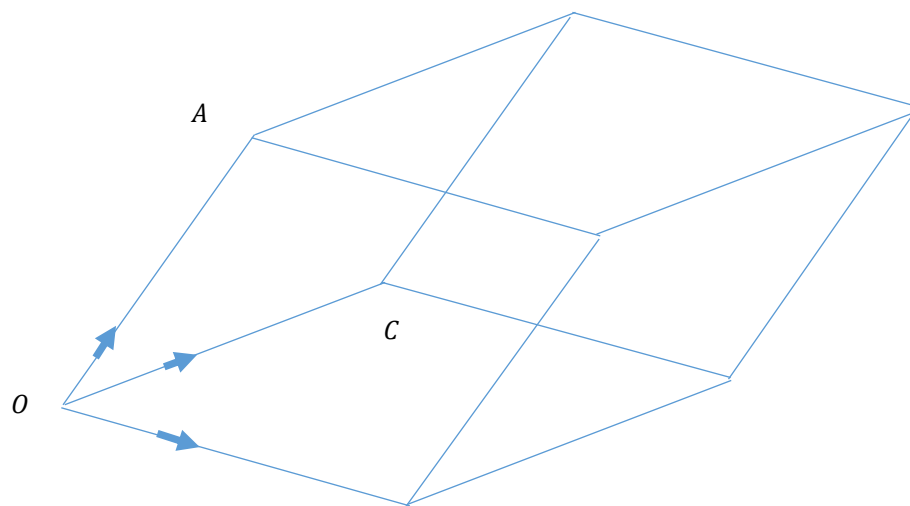


Figure-2

In figure 1 we can determine the quantity of the two objects by adding them, but in figure 2 we cannot determine such a quantity by adding extra information into an image.

The triple scalar product



$$\text{volume of the parallelepiped} = |(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a}|$$

Figure 3- The parallelepiped

The image vector

An image can be broken into pixels which are the basic components of an image. Each pixel has a lot of data and the objective was to use them.



Figure-4-Sample Image

0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	0,10
1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,10
2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	3,10
3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,9	4,10
4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	5,10
5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	6,10
6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	7,10
7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9	8,10
8,1	8,2	8,3	8,4	8,5	8,6	8,7	8,8	8,9	9,10
9,1	9,2	9,3	9,4	9,5	9,6	9,7	9,8	9,9	10,10

Figure-5- The Image with basic pixels

There are three main colors red, green and blue. Each pixel contains these colors in their respective quantities. As an example if we take a pixel from figure-4, we can project the pixel from 2-D to 3-D by using their RGB color quantity as shown in figure-5.

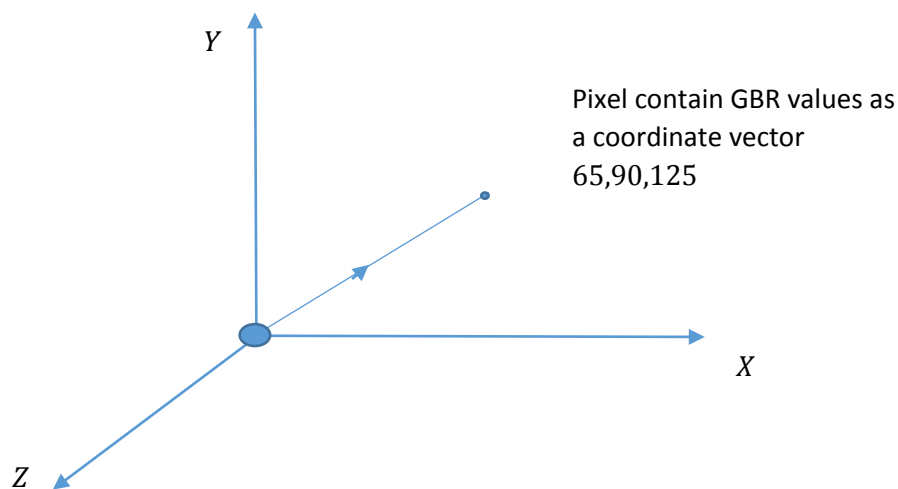


Figure-6

From the image in figure-4, take four pixels in order as follows and the four pixel represent their respective position vectors.

Take the position vectors as follows.

$\mathbf{a}_{1,1}$	$\mathbf{a}_{1,2}$	$\mathbf{a}_{1,3}$	$\mathbf{a}_{1,4}$
$\mathbf{a}_{2,1}$	$\mathbf{a}_{2,2}$	$\mathbf{a}_{2,3}$	$\mathbf{a}_{2,4}$
$\mathbf{a}_{3,1}$	$\mathbf{a}_{3,2}$	$\mathbf{a}_{3,3}$	$\mathbf{a}_{3,4}$

Figure-6- First four pixels on an image

Take the position vectors as follows.

$$\mathbf{a}_{i,j} = R_{i,j}\mathbf{i} + G_{i,j}\mathbf{j} + B_{i,j}\mathbf{k} \quad ; i, j = 1,2,3,4$$

Then take vectors,

$$\overrightarrow{A_1A_2} = \mathbf{a}_{1,2} - \mathbf{a}_{1,1}$$

$$\overrightarrow{A_1A_3} = \mathbf{a}_{1,3} - \mathbf{a}_{1,1}$$

$$\overrightarrow{A_1A_4} = \mathbf{a}_{1,4} - \mathbf{a}_{1,1}$$

Calculate the volume represented in the parallelepiped for the above three vectors,

$$V_1^1 = |(\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3}) \cdot \overrightarrow{A_1A_4}|$$

By repeating the procedure for the next set of four position vectors and calculate the respective volume,

$$V_2^1 = |(\overrightarrow{A_2A_3} \times \overrightarrow{A_2A_4}) \cdot \overrightarrow{A_2A_5}|$$

By applying this procedure to the raw 1 in figure 14, total volume of raw 1 has been derived as follows.

$$V_1 = \sum_{j=1}^{\text{\#column}-3} V_j^1$$

Then total volume has been derived as follows,

$$V_{total} = \sum_{i=1}^{row} V_i$$

The procedure for computing the difference in an image.

We can see clearly how the image is quantified by using the above proposed method. Then we can find the percentage difference of an image which is going to change with time.

In this, the volume of each row of an image is calculated and the difference between them are derived.

$$D_i = V_{i+1} - V_i \quad , i = 1, 2, \dots, \#row$$

Then calculating all the difference we derive,

$$D_{total} = \sum_{i=1}^{\#row-1} D_i$$

$$\text{The percentage difference of the image} = \frac{D_{total}}{V_{total}} \times 100$$

Conclusion.

We note that these results can be useful for in different fields such as medicine sector. In this way we can quantified the image volume and then identify at even small changes which occur is an image. That method shows the use of triple scalar product in practice.

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